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CREA Swiss Weakness Index: the Methodology*

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Abstract

In this technical note, we describe the Markov-switching model with heterogeneous recessions and expansions used to construct the economic weakness index for Switzerland. Given the non-linearities embedded in the model, we rely on Bayesian methods similar to Leiva-Leon et al. (2023) to solve the model. Estimated recession probabilities at cantonal level are then aggregated to obtain the economic weakness index for Switzerland.

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1 A Markov-Switching Model with Heterogeneous Recessions and Expansions

1.1 The Model

In each time period $t = 1, ..., T$, business cycle in canton c denoted by $b_{c,t}$ can be either in a regime of recession ($S_{c,t} = 1$) or expansion ($S_{c,t} = 0$). The $\tau_{1;c}$-th ($\tau_{0;c}$-th) cantonal recessionary (expansionary) episode is denoted by $\tau_{1;c} = 1, ..., N_{1;c}$ ($\tau_{0;c} = 1, ..., N_{0;c}$). We suppose that the dynamics of the business cycle is governed by the following Markov-switching model:

$$b_{c,t} = (1 - S_{c,t})\mu_{0,\tau_{0;c}} + S_t\mu_{1,\tau_{1;c}} + e_{c,t} \tag{1}$$

where $e_{c,t} \sim i.i.dN(0, \sigma^2_{e;c})$. The latent regime $S_{c,t} \in \{0, 1\}$ is assumed to follow a first-order Markov-switching process with the following transition probabilities:

$$Pr(S_{c,t} = 1|S_{c,t-1} = 1) = p_c \tag{2}$$

$$Pr(S_{c,t} = 0|S_{c,t-1} = 0) = q_c \tag{3}$$

This model follows Eo and Kim (2016) except that we assume as in Baumeister et al. (2022) that each episode is characterized by a specific mean that follows an iid Gaussian process:

$$\mu_{1,\tau_{1;c}} \sim i.i.dN(\overline{\mu}_{1,\tau_{1;c}}, \sigma^2_{1,\tau_{1;c}}) \tag{4}$$

$$\mu_{0,\tau_{0;c}} \sim i.i.dN(\overline{\mu}_{0,\tau_{0;c}}, \sigma^2_{0,\tau_{0;c}}) \tag{5}$$

The assumption allows means associated to recessionary and expansionary episodes to change very quickly in the presence of big shocks such as the Covid-19 shock. Given the non-linearities embedded in the model, we rely on Bayesian methods similar to Leiva-Leon et al. (2023) to estimate the model.

1.2 The Algorithm to Solve the Model

To ease notation, we drop cantonal index c. Let $Y = \{b_t\}_{t=1}^T$ contain all the available data on the cantonal business cycle index (BCI). Let $S = \{S_t\}_{t=1}^T$ be the collection of the latent regimes, and let $\mu = \{\mu_{0,t}, \mu_{1,t}\}_{t=1}^T$ contain the information on the regime-dependent means associated with recessionary and expansionary episodes. All the model parameters are collected in $\theta = \{\sigma^2_e, p, q\}$. Given data Y and prior distribution for θ, we rely on the Gibbs sampler to generate a sample of draws $\{S^{(j)}, \mu^{(j)}, \theta^{(j)}\}_{j=1}^J$ that represents the posterior density $p(S, \mu, \theta|Y)$. The following iterative procedure is used to generate the j-th draw:

Step 1. Draw $S^{(j)}$ from the density $p(S^{(j)}|\mu^{(j-1)}, \theta^{(j-1)}, Y)$ by using the simulation smoother proposed in Kim et al. (1999) that entails 2 steps.

Step 1.1. First, we compute $P_t(S_t = 1) \equiv Pr(S_t = 1|Y_t)$ that denotes the probability of being in a recession conditional on time-t available data $Y_t = \{b_t\}_{t=1}^{t}$:

$$\begin{aligned}
P_t(S_t = 1) &= \frac{Pr(S_t = 1, b_t, Y_{t-1})}{Pr(b_t, Y_{t-1})} \\
&= \frac{Pr(b_t|S_t = 1, Y_{t-1})Pr(S_t = 1|Y_{t-1})}{Pr(b_t|Y_{t-1})} \\
&= \frac{Pr(b_t|S_t = 1)Pr(S_t = 1|Y_{t-1})}{Pr(b_t|S_t = 1)Pr(S_t = 1|Y_{t-1}) + Pr(b_t|S_t = 0)Pr(S_t = 0|Y_{t-1})} \\
&= \frac{\phi\left(\frac{b_t - \overline{\mu}_{1,\tau_1}}{\sigma^2_{1,\tau_1}}\right) P_{t-1}(S_t = 1)}{\phi\left(\frac{b_t - \overline{\mu}_{1,\tau_1}}{\sigma^2_{1,\tau_1}}\right) P_{t-1}(S_t = 1) + \phi\left(\frac{b_t - \overline{\mu}_{0,\tau_0}}{\sigma^2_{0,\tau_0}}\right) P_{t-1}(S_t = 0)} \qquad (6)
\end{aligned}$$

where $\phi(\cdot)$ is the Gaussian probability density function. The probability of being in recession at time t conditional on Y_{t-1} is given by:

$$\begin{aligned}
P_{t-1}(S_t = 1) &= Pr(S_{t-1} = 1|Y_{t-1})Pr(S_t = 1|S_{t-1} = 1) + Pr(S_{t-1} = 0|Y_{t-1})Pr(S_t = 1|S_{t-1} = 0) \\
&= P_{t-1}(S_{t-1} = 1) \times p + P_{t-1}(S_{t-1} = 0) \times (1 - q)
\end{aligned}$$

We compute $P_t(S_t = 1)$ from $t = 1$ to T. We start the sequence with $P_0(S_1 = 1)$ being equal to its steady-state value.[1]

Step 1.2. Second, we compute $P_T(S_t = 1) \equiv Pr(S_t = 1|Y_T)$. To do so, we start the sequence by drawing S_T from $P_T(S_T = 1)$ given by (6). Then, given S_T, we draw S_{T-1} from the distribution $Pr(S_{T-1}|S_T, Y_{T-1})$. By repeating the process backwards for $t = T-2, ..., 1$, we have that $P_T(S_t = 1) = Pr(S_t = 1|S_{t+1}, Y_t)$. The posterior distribution for the recessionary state is given by:

$$\begin{aligned}
Pr(S_t = 1|S_{t+1}, Y_t) &= \frac{Pr(S_{t+1}, S_t = 1, Y_t)}{Pr(S_{t+1}, Y_t)} \\
&= \frac{Pr(S_{t+1}|S_t = 1, Y_t)Pr(S_t = 1|Y_t)}{Pr(S_{t+1}|Y_t)} \\
&= \frac{Pr(S_{t+1}|S_t = 1)Pr(S_t = 1|Y_t)}{Pr(S_t = 1|Y_t)Pr(S_{t+1}|S_t = 1) + Pr(S_t = 0|Y_t)Pr(S_{t+1}|S_t = 0)} \\
&= \frac{Pr(S_{t+1}|S_t = 1)P_t(S_t = 1)}{P_t(S_t = 1)Pr(S_{t+1}|S_t = 1) + P_t(S_t = 0)Pr(S_{t+1}|S_t = 0)}
\end{aligned}$$

[1] The steady-state value of the unconditional probability distribution of $S_t = 1$ is equal to $P_{t-1}(S_{ss} = 1) = \frac{1-q}{1-[p-(1-q)]}$.

Thus, given S_{t+1}, we draw S_t from:

$$
\begin{aligned}
Pr(S_t = 1|S_{t+1} = 1, Y_t) &= \frac{p \times P_t(S_t = 1)}{P_t(S_t = 1) \times p + P_t(S_t = 0) \times (1 - q)} \\
Pr(S_t = 1|S_{t+1} = 0, Y_t) &= \frac{(1 - p) \times P_t(S_t = 1)}{P_t(S_t = 1) \times (1 - p) + P_t(S_t = 0) \times q}
\end{aligned}
$$

Step 2. Draw $\mu^{(j)}$ from the density $p(\mu^{(j)}|S^{(j)}, \theta^{(j-1)}, Y)$. Given $S^{(j)}$, we partition the sample into expansionary episodes $\tau_0 = 1, ..., N_0$ and recessionary episodes $\tau_1 = 1, ..., N_1$. For each individual epsiode τ_κ of length T_{τ_κ} ($\kappa = \{0, 1\}$), we simulate the specific mean μ_{κ,τ_κ} from the posterior density $N(\overline{\mu}_{\kappa,\tau_\kappa}, \sigma^2_{\kappa,\tau_\kappa})$ derived from the combination of the the normal prior distribution $N(a_\kappa, b_\kappa)$ with the likelihood function, where :

$$
\sigma^2_{\kappa,\tau_\kappa} = \left(\frac{1}{b_\kappa} + \frac{T_{\tau_\kappa}}{\sigma^2_e}\right)^{-1} \tag{7}
$$

$$
\overline{\mu}_{\kappa,\tau_\kappa} = \sigma^2_{\kappa,\tau_\kappa}\left(\frac{a_\kappa}{b_\kappa} + \frac{\sum_{t\in\tau_\kappa} b_t}{\sigma^2_e}\right) \tag{8}
$$

Step 3. Draw $\theta^{(j)}$ from the density $p(\theta^{(j)}|S^{(j)}, \mu^{(j)}, Y)$ using the standard Gibbs sampling techniques.

Step 3.1. First, assuming a prior inverse-gamma distribution for $\sigma^2_e \sim IG(a, b)$, simulate σ^2_e from the posterior density that is also an inverse-gamma $IG(\overline{a}, \overline{b})$ with:

$$
\overline{a} = a + \frac{1}{2}T \tag{9}
$$

$$
\overline{b} = \left(b + \frac{1}{2}\sum_{t=1}^{T} e^2_t\right)^{-1} \tag{10}
$$

where $e_t = b_t - E(b_t)$ and $E(b_t) = (1 - S_t)\mu_{0,\tau_0} + S_t\mu_{1,\tau_1}$.

Step 3.2. Second, we assume a prior beta distribution $\beta(a, b)$ for both p and q. We draw p from the beta posterior density $\beta(\overline{a}_1, \overline{b}_1)$, where $\overline{a}_1 = a + n_{11}$ and $\overline{b}_1 = b + n_{10}$. n_{11} is the number of periods that the economy remains in recession from $t - 1$ to t, while n_{10} is the number of periods that the economy switches from a recession to an expansion. Similarly, we draw q from the beta posterior density $\beta(\overline{a}_0, \overline{b}_0)$, where $\overline{a}_0 = a + n_{00}$ and $\overline{b}_0 = b + n_{01}$.

Initial values and hyper-parameters of prior distributions. We select 12'000 draws. We drop the first 2'000 draws and keep the 10'000 remaining draws. We start the Gibbs sampler with $p = 0.8$ and $q = 0.9$. The initial values for the time series of the latent regime is equal to one if the trend of the BCI estimated with the HP filter is below zero. The initial values for the time series of the state-dependent mean are equal to prior mean (i.e. 1 for expansion and -1 for recession). The variance of the shock to the BCI is set to the prior variance of the state-dependent mean (i.e. $\sigma_2^e = 0$). The hyper-parameters associated with the prior distributions are reported in Table B1.

Table B1: Details of the prior distributions

Parameter	Meaning	Distribution	a	b
μ_{0,τ_0}	Mean during expansion	$N(a,b)$	1	0.1
μ_{1,τ_1}	Mean during recession	$N(a,b)$	-1	0.1
σ_2^e	Variance of shock to the BCI	$IG(a,b)$	3	$1{\times}(a-1)$
p	Probability of staying in an expansion	$\beta(a,b)$	9	1
q	Probability of staying in a recession	$\beta(a,b)$	8	2

Notes: The table reports the hyper-parameters associated with the prior distributions employed in the estimation algorithm.

2 The CREA Swiss Weekness Index

2.1 The Index

Since the models are estimated in a Bayesian fashion, we can generate many replications associated with the realization of recessionary episodes for each canton, that is, $S_{c,t}^{(j)}$ for $c = 1, ..., C$ and $j = 1, ..., J$, where $C = 26$ is the number of Swiss cantons and $J = 10'000$ is the number of retained draws. Consequently, the j-th replication of the SWI at time t is given by:

$$SWI_t^{(j)} = \sum_{c=1}^{C} \omega_{c,t} S_{c,t}^{(j)} \quad (11)$$

where $\omega_{c,t} \in [0,1]$ denotes the time-varying weight for each Swiss canton. These weights are based on the evolving economic size of each canton relative to national real GDP. The time-t Swiss Weekness Index is given by the median of the simulated density $\{SWI_t^{(j)}\}_{j=1}^{J}$.

2.2 Recession Probabilities at the Cantonal Level

The time-t recession probability in canton c is given by:

$$Pr(S_{c,t} = 1) = \frac{1}{J} \sum_{j=1}^{J} S_{c,t}^{(j)} \quad (12)$$

References

Baumeister, C., D. Leiva-León, and E. Sims (2022): "Tracking weekly state-level economic conditions," *Review of Economics and Statistics*, 1–45.

Eo, Y. and C.-J. Kim (2016): "Markov-switching models with evolving regime-specific parameters: Are postwar booms or recessions all alike?" *Review of Economics and Statistics*, 98, 940–949.

Kim, C.-J., C. R. Nelson, et al. (1999): "State-space models with regime switching: classical and Gibbs-sampling approaches with applications," *MIT Press Books*, 1.

Leiva-Leon, D., G. Pérez-Quirós, and E. Rots (2023): "Real-time weakness of the global economy," *Journal of Applied Econometrics*.